# RH 2010 <br> Hall measurement system 

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All mathematics that are used in the software of the RH 2010 Hall measurement system, can be calculated straight forward from basic physics or is published in the 50 th by van der Pauw (specific resistivity measurement) and Hall (Hall effect measurement).

All calculations and formulars are based on the following sample structur.

Sample structure.
Top view.


A square sample with 4 Ohmic contact at the edges.

The area of the contacts should be small compared to the total sample area (smaller that shown above). A circled sample with 4 contacts symetrically set on the boundary is also possible.

The specific resistivity Rho and the Hall coefficiant RH are evaluated from measured 4point resistivities at different contact configurations and RH at different magnetic fields ( RH ).
The 4-point resistivities are always be measured using V/I curves with preset currents using a variable current source and a voltage measurement circut with automatic amplification. The current source and the voltage measurement circut can be set using a special relais switch box to every contacts of the sample.
The V/I curves are automatically measured using 2 to $40 \mathrm{~V} / \mathrm{l}$ points (user definable). The 4-point resistivities are calculated from the slope of the V/I curves using linear regression. The regression section can be selected automatiaclly or manually. It will be saved with the measured data.

In the Hall (and v.d. Pauw) software menues and printouts the following definitions are used.

$$
\begin{array}{ll}
\text { I (contactpair) } & :=\text { current is set to defined contactpair } \\
\text { e.g. I(1,2) } & :=\text { Current is set to the contacts } 1 \text { and } 2 \text { of the sample. }
\end{array}
$$

V (contactpair) $\quad:=$ Voltage is measured at the defined contact pair e.g. $\quad V(3,4) \quad:=$ Voltage is measured at sample contacts 3 and 4

The resistivity is defined by: $R$ (contactpair I / contactpair V )

$$
\text { e.g } R(1,2 / 3,4) \text { : current contactpair }=1,2 \text {, voltage contactpair }=3,4
$$

If both contactpairs are the same ( 2-point measurement ) only one is given for the resistivity definition. e.g. $R(1,2)$ : 2-point resistivity beween the contacts 1 and 2. Typical results of a 2-point and 4-point V/I measurement are shown below.

2-point measurement








Using these definitions and the results of van der Pauw (Philips Res. Rep. 3 1958) the specific resistivity can be calculated out of two 4-point measurements:

$$
\begin{aligned}
& \text { Rho }=C^{*} D^{*} F^{*} 0.5^{*}(R(1,2 / 4,3)+R(2,3 / 1,4)) \\
& C=\pi / \operatorname{Ln}(2) \\
& F=\quad \begin{array}{l}
\text { Correction factor for unsysmetric samples with } \\
R(1,2 / 4,3)><R(2,3 / 1,4) \\
\text { (see publication })
\end{array} \\
& D=\begin{array}{l}
\text { Thickness of the sample or the layer }
\end{array}
\end{aligned}
$$

The software displays also the sheet resistivity $\mathrm{Rho}^{\prime}=\mathrm{Rho}$ at $\mathrm{D}=1 \mathrm{~cm}$

The correction factor F can not be given is a closed function. It is correctly calculated and put in a table as a function of the symetry factor Sym:
Sym: = R(1,2/4,3) / R(2,3/1,4)

All plots of van der Pauw results will show the values for the correction function and the symetry factor.

Two different v.d.Pauw configurations are possible (one is used above). Both configurations are automatically measured and Rho is calculated by averaging Rho1 and Rho 2. Offset voltages (s. below, 1. plot) caused by contacts or thermoelectric effects do not effect the values for the v.d. Pauw resistances.


For Hall measurements a contact configuration is used, that gives current and voltage perpendicular to each other. $R(1,3 / 2,4)$ or $R(2,4 / 1,3)$ as measurement configurations are possible and can be selected for the measurement. For ideal samples (ideal square symetry) the measured voltage at 0 magnetic field should be 0 independantly of the used current. This gives also 0 for the resistivity. Real samples will still give a voltage, the so called misalignement voltage. In some cases, if this misalignement voltage is much higher than the Hallvoltage, this voltage can cause big systematical errors. Therefore the RH 2010 Hallsystem contains a hardware compensatation of this misalignement voltage and gives much better results on asymetric samples as Hall systems without it. The compensation mode can be switched on by software. If the magnetic field is applied to the sample (perpendicular), the Hall voltage VH can be measured at the voltage contacts. For the configuration above $\mathrm{V}_{\mathrm{H}}$ is given by:

| $V_{H}=$ | $R_{H} * I * B * D^{-1}$ |
| ---: | :--- |
| $V_{H}$ | $=$ Hall voltage |
| $R_{H}$ | $=$ Hall coefficiant |
| $I$ | $=$ Measurement current |
| $D$ | $=$ Thickness of the sample or layer |
| $B$ | $=$ Magnetic field |

The Hall coefficiant RH contains the sample parameters (see below) and will be the result of a Hall measurement. Using the definitions above, we get:

$$
R_{H}=D^{*}(R(1,3 / 2,4), B=+B m a x)-(R(1,3 / 2,4, B=-B m a x)) /\left(2^{*} B m a x\right)
$$

The Hall measurements will be done always at magnetic fields symetrically to 0 . Bmax can be set by the user. The alternative contact configuration $R(2,4 / 1,3$, $B$ ) can also be selected by the user. Similar to the v.d.Pauw measurement the Hall resistivities $R$ ( $1,3 / 2,4, \mathrm{~B}$ ) are measured by $\mathrm{V} / \mathrm{I}$ curves and calculated by the slope, using linear regresssion.

A typical result of a Hall measurement is shown on the right.


The Hall coefficiant Rн can also be measured using more than 2 B-fields by measureing the Hall resistance as a function of the magnetic field. RH can be calculated then from the slope of the $R(1,3 / 2,4, B)$ versus $B$ plot by:

$$
\mathrm{R}_{\mathrm{H}}=\mathrm{D} * \Delta \mathrm{R}(1,3 / 2,4, B) / \Delta \mathrm{B}
$$

A typical result of this magnetic field dependance Hall measurement is shown below.


Each Hall resistance in the plot above is still measured by an V/I curve and calculated by automatic regression.

The measurement results Rho and RH can now be used for calculation of the sample parameters as carrier concentration, carrier type and carrier mobility. Starting with:

$$
\begin{aligned}
R h o & =\left(q^{*}\left(p+n^{*} b\right)^{*} \mu_{n}\right)^{-1} \\
R_{H} & =\left(p-n^{*} b^{2}\right) /\left(q^{*}\left(p+n^{*} b\right)^{2}\right.
\end{aligned}
$$

q : electroncharge
n : electronconcentration
p : holeconcentration
$\mu_{\mathrm{n}}$ : electron mobility
$\mu_{\mathrm{p}}$ : hole mobility
b $:=\mu_{\mathrm{n}} / \mu_{\mathrm{p}}$

For one carrier type material ( $n \gg p$ or $p \gg n$ ) the following approximations are valid:

$$
\begin{aligned}
\text { Rho } & =\left(q^{*} n^{*} \mu_{n}\right)^{-1} & \text { or } & \text { Rho }=\left(q^{*} p^{*} \mu_{p}\right)^{-1} \\
R_{H} & =-\left(q^{*} n\right)^{-1} & & \text { or } \quad R_{H} \quad=+\left(q^{*} p\right)^{-1}
\end{aligned}
$$

This leads to the following formulars, that are used for calculating the sample parameters out of the measurement results Rho and RH.

$$
\begin{aligned}
& \mathrm{n}, \mathrm{p}=\left(\mathrm{R}^{*} \mathrm{q}\right)^{-1} \quad-=\mathrm{n}, \quad+=\mathrm{p} \\
& \mu_{\mathrm{n}}, \mu_{\mathrm{p}} \quad=\mathrm{RH} / \mathrm{Rho} \quad-=\mu_{\mathrm{n}},+=\mu_{\mathrm{p}}
\end{aligned}
$$

The sign of $R_{\text {н }}$ is a measurement result. The software automatically transferes it into $n$ or $p$. These calculated values are also always displayed on a measurement result plot.

A complete combined v.d.Pauw and Hall measurement is shown below.
(A very symetrically sample with very good Ohmic contacts compared to that ones shown above)


A complete sample characterization including all 2-point, all 4-point, v.d. Pauw and Hall measurements are shown in the last two pages. The plots are as printed from our software. The first page shows a measurement @ two magnetic fields, the second one a measurement under variated magnetic field.

Name = LC_3@A_001.HTB
Name = LC_3@A_001.HTB
Comm $=$ High Res amplifier
ID = LC_3 rcID $=1001$
Date $=20 . \overline{06} .2000 \quad$ Type $=p-S i$
Thick. $=250.00$ um $\quad$ Temp $=295.00 \mathrm{~K}$
Field $=0.00 \mathrm{~T}$
$R(1,2)=6.89 \mathrm{E}+03 \quad \mathrm{R}(2,3)=7.73 \mathrm{E}+03 \mathrm{Ohm}$
$R(3,4)=3.88 \mathrm{E}+03 \quad R(4,1)=4.18 \mathrm{E}+03$ Ohm
$R(1,3)=4.59 \mathrm{E}+03 \quad \mathrm{R}(2,4)=7.60 \mathrm{E}+03$ Ohm




Voltage $[\mathrm{mV}] \rightarrow$
Voltage $[\mathrm{mV}] \rightarrow$
Name $=$ @A_001.HTB
Comm $=$ High Res amplifier
Name $=$ @A_001.HTB
Comm $=$ High Res amplifier
ID = LC_3
ID = LC_3
$\mathrm{rcID}=1001$
$\mathrm{rcID}=1001$
Date $=20.06 .2000$
Date $=20.06 .2000$
Type $=\mathrm{p}-\mathrm{Si}$
Type $=\mathrm{p}-\mathrm{Si}$
Thick. $=250.00 \mathrm{um}$
Thick. $=250.00 \mathrm{um}$
Temp $=295.00 \mathrm{~K}$
Temp $=295.00 \mathrm{~K}$
Field $=0.21 \mathrm{~T}$
Field $=0.21 \mathrm{~T}$
Rho $=3.94 \mathrm{E}+01 \mathrm{Ohm}^{*} \mathrm{~cm}$
Rho $=3.94 \mathrm{E}+01 \mathrm{Ohm}^{*} \mathrm{~cm}$
$R h o^{\prime}=1.58 \mathrm{E}+03 \mathrm{Ohm}$
$R h o^{\prime}=1.58 \mathrm{E}+03 \mathrm{Ohm}$
Sym $=5.01 \mathrm{E}+00$
Sym $=5.01 \mathrm{E}+00$
$\mathrm{F}=8.21 \mathrm{E}-01$
$\mathrm{F}=8.21 \mathrm{E}-01$
$\leftarrow[\wedge \omega]$, әбено $\wedge$
$\leftarrow[\wedge \omega]$, әбено $\wedge$
$R h=1.16 \mathrm{E}+04 \mathrm{~cm}^{3} / \mathrm{As}$
$R h=1.16 \mathrm{E}+04 \mathrm{~cm}^{3} / \mathrm{As}$
$\mathrm{Rh}^{\prime}=4.67 \mathrm{E}+05 \mathrm{~cm}^{2} / \mathrm{As}$
$\mathrm{Rh}^{\prime}=4.67 \mathrm{E}+05 \mathrm{~cm}^{2} / \mathrm{As}$
$\mathrm{p}=5.34 \mathrm{E}+14 \mathrm{~cm}^{-3}$
$\mathrm{p}=5.34 \mathrm{E}+14 \mathrm{~cm}^{-3}$
my $=2.96 \mathrm{E}+02 \mathrm{~cm}^{2} \mathrm{Ns}$
my $=2.96 \mathrm{E}+02 \mathrm{~cm}^{2} \mathrm{Ns}$

Name = @A_001.HTB
Comm = High Res amplifier
ID = LC_3
rcID $=1001$
Date $=20.06 .2000$
Type $=\mathrm{p}-\mathrm{Si}$
Thick. $=250.00$ um
Temp $=295.00 \mathrm{~K}$
Field $=0.00 \mathrm{~T}$
$R(1,2)=6.89 E+03$
$R(2,3)=7.73 \mathrm{E}+03$
$R(3,4)=3.88 \mathrm{E}+03$
$R(4,1)=4.18 \mathrm{E}+03$
$R(1,3)=4.59 \mathrm{E}+03$
$\mathrm{R}(2,4)=7.60 \mathrm{E}+03$
$R(1,2 / 4,3)=1.40 \mathrm{E}+02$
$\mathrm{R}(2,3 / 1,4)=7.08 \mathrm{E}+02$
$\mathrm{R}(3,4 / 2,1)=1.41 \mathrm{E}+02$
$\mathrm{R}(4,1 / 3,2)=7.05 \mathrm{E}+02$
$R(1,3 / 2,4)=5.65 \mathrm{E}+02$
$R(2,4 / 3,1)=-5.68 \mathrm{E}+02$
Voltage ${ }^{\text {[ }}[\mathrm{mV}] \rightarrow$



